



Math Tracks™: Investing & Spending

On this Math Track™, you're going to think about money: Investing & Spending. The goal is to learn about some basic finance concepts including:

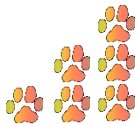
- Simple Interest
- Compound Interest
- Loans (car & home)
- Investments (lump sum & periodic)

You will also learn how to use the graphing calculator (we use the TI-83+).

Items Needed:

- ⇒ Graphing calculator
- ⇒ Internet access or good finance textbook to use as a reference
- ⇒ Paper & pencil to record your answers

Award Levels



Award Level	Points Required
Junior Adventurer	60
Master Tracker	80
Expert Explorer	100

Investing & Spending Activity

Chris and Kim graduate at age 22 in December 2010. Both begin jobs in January 2011. Kim immediately begins saving \$200 per month and continues for 10 years then stops and lets the money draw interest until age 65 (6% compounded monthly). Instead of starting a savings plan, Chris buys a sports car. At age 45, Chris begins saving \$200 per month and continues until age 65 (6% compounded monthly).

Questions: (10 points each)

1. How much of Kim's own money was invested?
2. How much of Chris's own money was invested?
3. How much money did Kim have at the end of the contribution period (after 10 years)?
4. How much money did Kim have at age 65?
5. How much money did Chris have at age 65?
6. What do you think about this situation?
7. How much money would Kim have if the contributions did not stop after 10 years? In other words, how much money would Kim have if contributions were made monthly from age 22 to age 65?
8. Suppose Kim received an inheritance upon graduation. What lump sum amount should be invested in order to have the same amount of money in ten years as in #3.
9. Can you solve this problem in another way? Explain.
10. Chris finances \$30,000 for a sports car. What are the monthly payments if the interest rate is 9% annually for 5 years?
11. How much interest did Chris pay?

Formulas & Calculator Instructions¹, Page 1

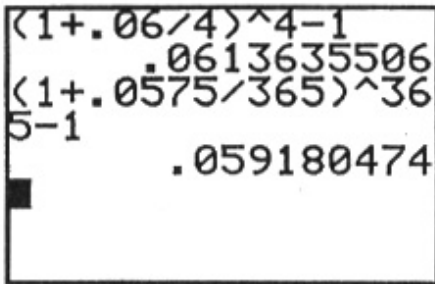
A Look at Investing (and Spending)

Effective annual rate

When looking at interest rates, it is really hard to determine which rate is better when there are different compounding periods. The way to accurately compare different rates with different compounding periods is to calculate the effective annual rate (or effective annual yield). This is the equivalent simple interest rate.

$$i_{\text{eff}} = \left(1 + \frac{r}{k}\right)^k - 1 \quad \text{where} \quad \begin{array}{l} r \text{ is the annual percentage rate} \\ k \text{ is the number of compounding periods} \end{array}$$

Example: Compare the effective annual rates of an account paying 6% compounded quarterly with an account paying 5.75% compounded daily.



The image shows a calculator screen with two lines of input and their corresponding results. The first line shows the calculation $(1 + .06/4)^4 - 1$ resulting in $.0613635506$. The second line shows the calculation $(1 + .0575/365)^365 - 1$ resulting in $.059180474$.

The 6% compounded quarterly will return the same as a simple interest rate of 6.14% while the 5.57% compounded daily will return the same as a simple interest rate of 5.92%.

A one time fixed amount investment

If you have a lump sum of money that you want to invest, you would use the formula for compound interest to determine the total amount of your investment after t years at $r\%$ interest rate compounded a specific number of times (k) a year.

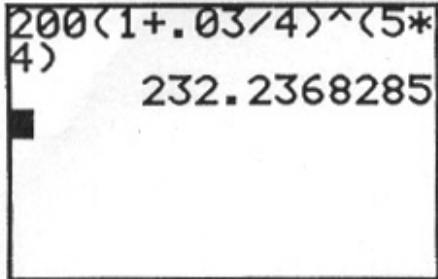
Compound Interest: $S = P\left(1 + \frac{r}{k}\right)^n$ where

- S is the total amount
- P is the initial principal invested
- r is the annual percentage rate
- k is the number of compounding periods annually
- $\frac{r}{k}$ is the interest rate per period
- n is the total number of compounding periods ($n = t \cdot k$)
- t is the number of years

¹ Calculator instructions from Texas Instruments.

Formulas & Calculator Instructions², Page 2

Example: \$200 compounded quarterly (4 times a year) for 5 years at 3% interest would amount to :



200(1+.03/4)^(5*4)
232.2368285

You will have \$232.24 at the end of 5 years.

Periodic contributions to a plan

If you cannot invest a lump sum, but want to systematically and faithfully invest a certain amount you would be contributing to an ordinary annuity. The amount of money accrued over a period of time is called the **future value** of an ordinary annuity. The future value of an ordinary annuity is found using the following formula:

$$S = R \frac{(1+i)^n - 1}{i}$$

where

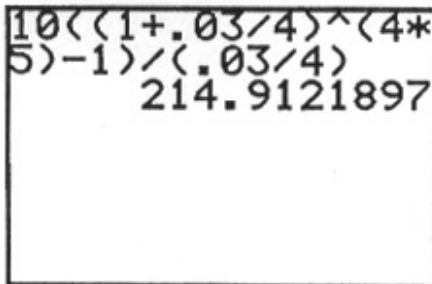
S is the future value (the amount of money accumulated)

R is the regular installment (a certain amount monthly, quarterly, etc.)

i is the interest rate per period, i.e., $i = \frac{r}{k}$ (same as above)

n is the number of compounding periods ($n = t \cdot k$)

Example: You do not have \$200 to invest at one time, but you can invest \$10 quarterly at 3% interest for five years (a total of \$200). How much money will you have?



10((1+.03/4)^(4*5)-1)/(.03/4)
214.9121897

You will have \$214.91 at the end of 5 years.

² Calculator instructions from Texas Instruments.

Formulas & Calculator Instructions³, Page 3

Borrowing money and paying it back

When you want to buy something on the installment plan and you know how much you can afford monthly, the current interest rate, and how long you are willing to pay on the item you need to know the **present value** of an ordinary annuity. That is, you borrow a certain amount now and pay it off over time. The present value of an ordinary annuity is found using the following formula:

$$A = R \frac{1 - (1 + i)^{-n}}{i}$$

where

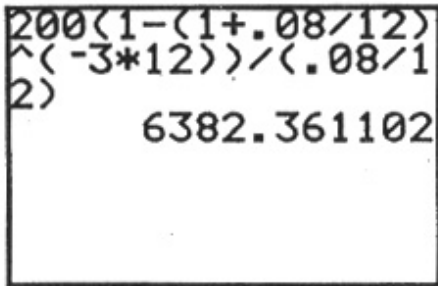
A is the present value of the annuity

R is the regular installment (a certain amount monthly, quarterly, etc.)

i is the interest rate per period, i.e., $i = \frac{r}{k}$ (same as above)

n is the number of compounding periods ($n = t \cdot k$)

Example: You need a new car. You have analyzed your income and have determined that you can afford a car payment of \$200 a month. By doing some research in the newspapers, you discover that used cars currently are financed at 8% interest. How much can you finance if you want the loan to be paid off in three years.



200(1-(1+.08/12)
^(-3*12))/(.08/1
2)
6382.361102

You can afford to finance \$6382.36.

³ Calculator instructions from Texas Instruments.